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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, MAY 2016 THIRD YEAR [BATCH 2013-16]

Paper: VII-B Full Marks: 30

Group - B

Answer any three of the following:

 $[3\times10]$

- 1. a) What is phase space? A classical particle is free to move in a cube of side L. If its energy $\leq E$, find the volume of phase space available to it. [1+3]
 - b) Consider a free Fermi gas in two dimension, confined to a square area $A = L^2$
 - i) Derive a formula for the density of states.
 - ii) Find the Fermi energy (\in_F) and show that the average energy of the particles is $\frac{\in_F}{2}$ at absolute zero. [2+2+2]
- 2. a) State the classical equipartition theorem or energy. Comment on its limit of validity. [1+1]
 - b) Because of the Doppler effect, the frequency of the light observed from an atom having an x-component of velocity v_x is $v \simeq v_0 \left(1 + \frac{v_x}{c}\right)$. v_0 is constant.
 - i) Calculate <v>
 - ii) Calculate the dispersion $<(\Delta v)^2>=<(v-< v>)^2>$.
 - iii) Calculate the line width = $\sqrt{\langle (\Delta v)^2 \rangle}$ [3]
 - c) For a canonical system, utilize the Shanon's definition of entropy, to show that $S = \frac{\langle E \rangle}{T} + k_B T \ln Z \text{ and } F = k_B T \ln Z.$

Hence show that $G = F + PV = k_B T \left[-\ln Z + V \left(\frac{\partial \ln Z}{\partial V} \right) T \right]$ [symbols are of usual significance] [2+1+2]

- 3. a) Obtain an expression for the BE statistics using microcanonical ensemble. What is the range of the chemical potential in the BE statistics? [3+1]
 - b) Starting from the expression for B-E statistics, obtain Planck's law of energy distribution for photons. Show how Raybeigh-Jean's law and Wein's distribution law follow from Planck's law as limiting cases.

 [3+3]
- 4. a) Write down he expression of grand canonical partition function Z. Find the expression of mean energy <F>. [1+2]
 - b) Show that relative mean square fluctuation in the particle density (n), i.e., $\frac{\langle (\Delta n)^2 \rangle}{\langle n^2 \rangle} = \frac{k_B T}{V} K_T$, where K_T is the isothermal compressibility of the system and V is the volume of the system. [4]
 - c) 4 bosons are distributed among two energy levels (∈ and 2 ∈). Higher energy level is 2-fold degenerate. Find the number of microstates for which the total energy of the system is 7 ∈.
- 5. a) A drunk person starts from the origin and moves randomly along x or -x direction. Each step is of equal length ℓ . The probability that any one steps along x or along -x are same and equal to $\frac{1}{2}$. Suppose that the person has taken N steps and mean time taken for each step is τ .

- i) Estimate the probability $P'_{N}(r)$ that the displacement (x) of the person from the origin is equal to $r\ell$ (r is any positive integer). [2]
- ii) Find out the value < $r^2 >$. Show that < $r^2 >$ is proportional to t, where $t = N\tau$ is the time taken by the person for N steps. [2+1]
- b) Write down the distribution law obeyed by electron gas in metal and apply it to derive Richardson equation for thermionic emission. [5]

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